



Mathematics: Specialist Formula sheet Units 3A and 3B

Vectors

$$|(a, b)| = \sqrt{a^2 + b^2} \qquad |\mathbf{a} + \mathbf{b}| \leq |\mathbf{a}| + |\mathbf{b}|$$

$$\mathbf{a} \cdot \mathbf{b} = |\mathbf{a}| |\mathbf{b}| \cos \theta = a_1 b_1 + a_2 b_2$$

Vector equation of a line in the plane:

one point and the slope: $\mathbf{r} = \mathbf{r}_1 + \lambda \mathbf{l}$

two points: $\mathbf{r} = \mathbf{r}_1 + \lambda (\mathbf{r}_2 - \mathbf{r}_1)$

Vector form of the equation of a circle in the plane: $|\mathbf{r} - \mathbf{d}| = \rho$

Trigonometry

In any triangle ABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$\text{Area} = \frac{1}{2} ab \sin C$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

In a circle of radius r , for an arc subtending angle θ (radians) at the centre:

$$\text{Length of arc} = r\theta \qquad \text{Area of sector} = \frac{1}{2} r^2 \theta \qquad \text{Area of segment} = \frac{1}{2} r^2 (\theta - \sin \theta)$$

$$\sin (\theta \pm \phi) = \sin \theta \cos \phi \pm \cos \theta \sin \phi$$

$$\sin 2\theta = 2 \sin \theta \cos \theta$$

$$\cos (\theta \pm \phi) = \cos \theta \cos \phi \mp \sin \theta \sin \phi$$

$$\cos 2\theta = \cos^2 \theta - \sin^2 \theta = 2 \cos^2 \theta - 1 = 1 - 2 \sin^2 \theta$$

$$\tan (\theta \pm \phi) = \frac{\tan \theta \pm \tan \phi}{1 \mp \tan \theta \tan \phi}$$

$$\tan 2\theta = \frac{2 \tan \theta}{1 - \tan^2 \theta}$$

Exponentials and logarithms

For $a, b > 0$ and m, n real,

$$a^m a^n = a^{m+n}$$

$$a^m b^m = (ab)^m$$

$$(a^m)^n = a^{mn}$$

$$a^{-m} = \frac{1}{a^m}$$

$$\frac{a^m}{a^n} = a^{m-n}$$

$$a^0 = 1$$

For m an integer and n a positive integer: $\frac{m}{a^n} = \sqrt[n]{a^m} = \left(\sqrt[n]{a}\right)^m$

For $a, y > 0$, $x = \log_a y \Leftrightarrow y = a^x$

$$\log_a 1 = 0$$

$$\log_a a = 1$$

$$\log_a cd = \log_a c + \log_a d$$

$$\log_a (c^b) = b \log_a c$$

Functions

Differentiation

If $f(x) = y$, then $f'(x) = \frac{dy}{dx}$

If $f(x) = x^n$, then $f'(x) = nx^{n-1}$

If $f(x) = e^x$, then $f'(x) = e^x$

If $f(x) = \ln x$, then $f'(x) = \frac{1}{x}$

	Function notation		Leibniz Notation	
	y	y'	y	y'
Product rule	$f(x) g(x)$	$f'(x) g(x) + f(x) g'(x)$	uv	$\frac{du}{dx} v + u \frac{dv}{dx}$
Quotient rule	$\frac{f(x)}{g(x)}$	$\frac{f'(x) g(x) - f(x) g'(x)}{(g(x))^2}$	$\frac{u}{v}$	$\frac{\frac{du}{dx} v - u \frac{dv}{dx}}{v^2}$
Chain rule	$f(g(x))$	$f'(g(x)) g'(x)$	$y = f(u)$ and $u = g(x)$	$\frac{dy}{du} \times \frac{du}{dx}$

Integration

$\int x^n dx = \frac{x^{n+1}}{n+1} + c \quad n \neq -1$

$\int e^x dx = e^x + c$

$\int \frac{1}{x} dx = \ln x + c$

Fundamental Theorem of Calculus: $\frac{d}{dx} \int_a^x f(t) dt = f(x)$ and $\int_a^b f'(x) dx = f(b) - f(a)$

Piece-wise defined functions

Absolute value function: $|x| = \begin{cases} x & x \geq 0 \\ -x & x < 0 \end{cases}$

Sign function: $\text{sgn}(x) = \begin{cases} -1 & x < 0 \\ 0 & x = 0 \\ 1 & x > 0 \end{cases}$

Greatest integer function: $\text{int}(x) = \text{greatest integer} \leq x$ for all x

Measurement

Trapezium: Area = $\frac{1}{2} (a + b) \times \text{height}$, where a and b are the lengths of the parallel sides

Prism: Volume = Area of base \times height

Cylinder: Total surface area = $2\pi r h + 2\pi r^2$ Volume = $\pi r^2 \times h$

Pyramid: Volume = $\frac{1}{3} \times \text{area of base} \times \text{height}$

Cone: Total surface area = $\pi r s + \pi r^2$, s is the slant height Volume = $\frac{1}{3} \times \pi r^2 \times h$

Sphere: Total surface area = $4\pi r^2$ Volume = $\frac{4}{3} \pi r^3$

Note: Any additional formulas identified by the examination panel as necessary will be included in the body of the particular question.